Exercise 80

If the equation of motion of a particle is given by $s = A\cos(\omega t + \delta)$, the particle is said to undergo *simple harmonic motion*.

(a) Find the velocity of the particle at time t.

(b) When is the velocity 0?

Solution

The velocity is the derivative of the displacement function.

$$\begin{aligned} v(t) &= \frac{ds}{dt} \\ &= \frac{d}{dt}[A\cos(\omega t + \delta)] \\ &= A\frac{d}{dt}[\cos(\omega t + \delta)] \\ &= A\left[-\sin(\omega t + \delta)\cdot\frac{d}{dt}(\omega t + \delta)\right] \\ &= A\left[-\sin(\omega t + \delta)\cdot(\omega)\right] \\ &= -A\omega\sin(\omega t + \delta) \end{aligned}$$

It's zero whenever the argument of sine is an integer multiple of π,

$$\omega t + \delta = n\pi, \quad n = 0, \pm 1, \pm 2,$$

that is,

$$t = \frac{n\pi - \delta}{\omega}.$$